Lithological and surface geometry joint inversions using multi-objective global optimization methods

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Geophysical inversion primer

Forward problem

Earth model (e.g. density)

Inverse problem

Survey data (e.g. gravity)
Typical numerical approach for mesh-based inversion (underdetermined)

- Local descent-based minimization
- Weighted aggregate objective function

$$\min_{\mathbf{m}} f_a(\mathbf{m}) = \sum_i w_i f_i(\mathbf{m})$$

$$f_d = \frac{1}{N} \sum_{j=1}^{N} \left( \frac{d_{j}^{\text{pred}}(\mathbf{m}) - d_{j}^{\text{obs}}}{\sigma_j} \right)^2$$

$$f_a(\mathbf{m}) = \sum_i \lambda_i f_{d,i}(\mathbf{m}_i) + \sum_j \alpha_j f_{m,j}(\mathbf{m}_j) + \sum_k \rho_k f_{c,k}(\mathbf{m}_{k,1}, \mathbf{m}_{k,2})$$

$$\min_{\mathbf{m}} f_a(\mathbf{m}) = \lambda f_d(\mathbf{m}) + f_m(\mathbf{m})$$
Typical numerical approach for mesh-based inversion (underdetermined)

- Local descent-based minimization
- Weighted aggregate objective function

\[ \min_{m} f_a(m) = \sum_{i} w_i f_i(m) \]  

\[ f_d = \frac{1}{N} \sum_{j=1}^{N} \left( \frac{d_{j}^{\text{pred}}(m) - d_{j}^{\text{obs}}}{\sigma_j^2} \right)^2 \]  

\[ f_{a}(m) = \sum_{i} \lambda_i f_{d,i}(m_i) + \sum_{j} \alpha_j f_{m,j}(m_j) \]  

\[ + \sum_{k} \rho_k f_{c,k}(m_{k,1}, m_{k,2}) \]  

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Disadvantages of the typical inversion approach

1. Appropriate weights must be determined for the aggregate objective function.

\[ f_a(m) = \sum_i \lambda_i f_{d,i}(m_i) + \sum_j \alpha_j f_{m,j}(m_j) \]

\[ + \sum_k \rho_k f_{c,k}(m_{k,1}, m_{k,2}) \]
Disadvantages of the typical inversion approach

1. Appropriate weights must be determined for the aggregate objective function.
2. All objective functions must be differentiable.

\[ f(x) = \begin{cases} 
  x^2 & \text{if } x > 0 \\
  -x & \text{if } x < 0 \\
  0 & \text{if } x = 0 
\end{cases} \]
Disadvantages of the typical inversion approach

1. Appropriate weights must be determined for the aggregate objective function

2. All objective functions must be differentiable

3. Local minima entrapment may occur, providing suboptimal solutions
Disadvantages of the typical inversion approach

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Pareto Multi-Objective Global Optimization (PMOGO)

- Multi-Objective Problem (MOP):

\[
\min_{\mathbf{m}} \mathbf{f}(\mathbf{m}) = \left[ f_1(\mathbf{m}), f_2(\mathbf{m}), \ldots, f_L(\mathbf{m}) \right]
\]

- Concept of dominance:

\[
f_i(\mathbf{m}_a) \leq f_i(\mathbf{m}_b) \quad \text{for } i = 1, \ldots, L
\]

\[
f_i(\mathbf{m}_a) < f_i(\mathbf{m}_b) \quad \text{for at least one } i
\]

- We want to converge to the Pareto-optimal curve/surface (related to Tikhonov curve)
- Non-dominated Sorting Genetic Algorithm (NSGA-II) of Deb et al. (2002)
Massive sulphide deposit scenario from Carter-McAuslan et al. (2015)

Rock types:
- background (gneiss)
- intrusive (troctolite)
- lens (sulphide)
Joint inversion of gravity data and traveltimes

Aggregate objective function:

\[ f_a(m_1, m_2) = \left[ \lambda_1 f_{d1}(m_1) + \lambda_2 f_{d2}(m_2) \right] + \left[ f_{m1}(m_1) + f_{m2}(m_2) \right] + \rho f_c(m_1, m_2) \]

Joint coupling term (fuzzy c-means):

\[ f_c(m_1, m_2) = \sum_{i=1}^{C} \sum_{k=1}^{M} \sum_{j=1}^{N} w_{ik}^2 z_{ik}^2 \]

\[ z_{ik}^2 = (m_{1,k} - u_{1,i})^2 + (m_{2,k} - u_{2,i})^2 \]
Independent inversion results (aggregate & local minimization)

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Joint inversion with careful local minimization (hand holding)

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Joint inversion with careless local minimization

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Lithological and surface geometry joint inversions using multi-objective global optimization methods
Introduction

- Sulphide example

Results from typical strategy

Results from PMOGO strategy

Conclusion

Lithological inversion

- Define several a priori rock types (3 here)

- Define physical property distributions for each rock type (homogeneous here)

- PMOGO inversion assigns a rock type to each mesh cell

- Simple to add numerically complicated topological constraints, e.g. the model must contain:
  - 4 contiguous regions
  - all 3 rock types
  - 2 regions corresponding to the background

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Lithological inversion: 3D Pareto front

Pareto surface vs curve
Lithological inversion: 3D Pareto front

[Graphs showing the relationship between regularization and seismic and gravity misfit for different strategies.]
Lithological inversion: three models in the Pareto front

Misfits:
(a) expected targets
(b) high gravity
(c) high seismic

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Lithological inversion: some rudimentary statistics

Rock types:
(a) background
(b) intrusive
(c) sulphide

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Surface-based inversion

- Parameterization defines interfaces between rock units
- No mesh required (very fast forward problem)
- PMOGO inversion moves vertices
Surface-based inversion: 3D Pareto front

Results from typical strategy
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Surface-based inversion: 3D Pareto front

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Surface-based inversion: three models in the Pareto front

Legend:
- - feasible region
— true interfaces
— expected target misfits
— both misfits high
— high seismic misfit

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Advantages and disadvantages of PMOGO

Advantages:

• Obviate the requirement of having to deal with trade-off parameters
  → Joint inversion greatly simplified

• Automatically provide a suite of solutions across the Pareto front
  → Opportunities to calculate statistics

• Any numerically complicated objective functions or constraints can be used

• Avoid local minima entrapment
  → Fundamentally different model parameterizations

• Easy to parallelize

Disadvantages:

• Increased computing time (100-300x for mesh-based inversions, WITH CAVEATS)
Bijani et al., 2015, Three-dimensional gravity inversion using graph theory to delineate the skeleton of homogeneous sources. Geophysics, 80, G53–G66
